Math 1206 Formula Sheet

S. Rodney, Cape Breton University Mathematics

Integration Formulas:

1. Trig Identities: $\sin^2(x) + \cos^2(x) = 1,\ 1 + \tan^2(x) = \sec^2(x),\ \sin(2x) = 2\sin(x)\cos(x)$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2},\ \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

2. The derivatives of Inverse Trig functions

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}},\ \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2},\ \frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

3. Integration by parts:

$$\int u\ dv = uv - \int v\ du, \quad \int_a^b u\ dv = uv\Big|_a^b - \int_a^b v\ du$$

4. Some elementary integrals

$$\int x^r\ dx = \frac{1}{1+r}x^{1+r} + C;\ r \neq -1, \quad \int \sin(ax)\ dx = -\frac{1}{a}\cos(ax) + C, \quad \int \cos(ax)\ dx = \frac{1}{a}\sin(ax) + C$$

$$\int e^{ax}\ dx = \frac{1}{a}e^{ax} + C, \quad \int \frac{1}{x}\ dx = \ln|x| + C$$

Convergence Tests:

1. The integral test: Suppose that $a_n = f(n)$, where $f(x)$ is positive, continuous, and decreasing on the interval $[1, \infty)$. Then, each of

$$\int_1^\infty f(x)\ dx \text{ and } \sum_{n=1}^{\infty} a_n$$

either both converge or both diverge.

2. The comparison test: Let $\{a_n\}$ and $\{b_n\}$ be non-negative sequences for which $a_n \leq b_n$. If $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$. If $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

3. Geometric Series: If $-1 < r < 1$ then $\displaystyle\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$.