Math 1206 Final Exam

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True - False Problems.
Circle "T" if the statement is true, and "F" if it is false.

1.	Every function is equal to its Taylor series.	T	F
2.	If $f(x)$ is continuous on $[a,b]$ then it is always true that $\left\|\int_a^b f(x)\,dx\right\| < \infty$.	T	F
3.	Every absolutely convergent series converges.	T	F
4.	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges for $p = 1/3$.	T	F

Long Answer Section
Show All Of Your Work

1. Evaluate each of the following integrals:

$$(a) \int \frac{2x}{2x^2+3}\,dx, \quad (b) \int x\cos(x)\,dx, \quad (c) \int x\arctan(x)\,dx$$

$$(d) \int \frac{1}{x^2\sqrt{9+x^2}}\,dx \quad (\text{Use } x = 3\tan(\theta)), \quad (e) \int \frac{x^3}{x^2-x-2}\,dx$$

2. Evaluate each of the definite integrals.

$$(a) \int_0^{\pi/2} \frac{\cos(x)}{(1-\sin(x))^{1/3}}\,dx, \quad (b)\ e \int_e^{\infty} \frac{1}{x\sqrt{\ln(x)}}\,dx$$

3. Show all of your work and determine if the following series converge or diverge using the test of your choice.

$$(a) \sum_{n=1}^{\infty} \frac{n+6}{n\sqrt{n}}, \quad (b) \sum_{n=1}^{\infty} \frac{2^n 3^n}{n!}, \quad \text{and} \quad (c) \sum_{n=1}^{\infty} \frac{e^n}{3!}$$

4. (a) Find the Taylor series for $f(x) = \cos(\pi x)$ centred at $x = 2$.
   (b) Find the radius of convergence and interval of convergence for the power series you found.
   (c) Why is it advantageous to work with the Taylor series of a given function?

5. (a) Show that the alternating series

$$S = \sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3n}{3n^2-1}\right)$$

   converges.

6. Consider the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^3} - \frac{1}{(n+1)^3}\right)$

   (a) Find a formula for the k^{th} partial sum S_k for any positive integer k. (Hint: When you expand the partial sum, there will be lots of cancellation!)
   (b) Using the formula you found, find the sum of the series.

7. (a) Using the ideas of differentiation of power series, find a series representation for $f(x) = \dfrac{1}{(1-x)^3}$ centered at $x = 0$.
   (b) Find the interval of convergence for the series you found.

8. It is easy to see that

$$\int_0^1 \frac{4}{1+x^2}\,dx = \pi.$$

(a) How large should N be so that the Midpoint Rule approximation for this integral is within 10^{-10}. DO NOT FIND THE APPROXIMATION. Notice that this gives an approximation of π to 10 decimal places.
(b) Use Simpson's Rule with $N = 4$ to approximate the integral. How does your answer compare to π?

9. Using the washer method, find a formula for the volume of a cylinder of length h and radius r.

10. A exterior tank has a liner with a shape given by $f(x) = \sqrt{x}$ (x measured in meters) over the interval $[0, 9]$ spun around the x-axis.
    (a) What is the maximum volume of the tank?
    (b) Recalling that 1 cubic meter of space can contain 1000L of fluid, if gasoline costs 1.87 per litre, how much would it cost to fill the tank from empty?

11. Evaluate the integral:

$$\int \frac{(2e^x + 1)e^x}{(e^x + 1)(1 - e^x)}\,dx.$$

(Hint: Try $u = e^x$ to start)