Math 1206 Formula Sheet

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Integration Formulas:

1. Trig Identities: $\sin^2(x)+\cos^2(x)=1,\ 1+\tan^2(x)=\sec^2(x),\ \sin(2x)=2\sin(x)\cos(x)$

$$\sin^2(x)=\frac{1-\cos(2x)}{2},\ \cos^2(x)=\frac{1+\cos(2x)}{2}$$

2. The Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}(x)=\frac{1}{\sqrt{1-x^2}},\ \frac{d}{dx}\tan^{-1}(x)=\frac{1}{1+x^2},\ \frac{d}{dx}\sec^{-1}(x)=\frac{1}{|x|\sqrt{x^2-1}}$$

3. Integration by parts: $\int u\ dv=uv-\int v\ du,\quad \int_a^b u\ dv=uv\Big|_a^b-\int_a^b v\ du$

4. Some elementary integrals

$$\int x^r\ dx=\frac{1}{1+r}x^{1+r}+C;\ r\neq -1,\quad \int \sin(ax)\ dx=-\frac{1}{a}\cos(ax)+C,\quad \int \cos(ax)\ dx=\frac{1}{a}\sin(ax)+C$$

$$\int e^{ax}\ dx=\frac{1}{a}e^{ax}+C,\quad \int \frac{1}{x}\ dx=\ln|x|+C$$

5. Points of interest for Midpoint Rule: $m_j=a+(j-1/2)\Delta x,\ j=1..N$
   Points of Interest for Simpson's Rule: $x_j=a+j\Delta x,\ j=0..N$
   $\Delta x=\frac{b-a}{N}$.

6. Approximation errors:
   (a) MidPoint Rule: $\Delta_{M_n}\leq\frac{(b-a)^3K}{24n^2};\ |f''(x)|\leq K$
   (b) Simpson's Rule: $\Delta_{S_n}\leq\frac{(b-a)^5M}{180n^4};\ |f^{(IV)}(x)|\leq M$

7. Integrals involving $\sec(x)$:

$$\int \sec(x)\ dx=\ln|\sec(x)+\tan(x)|+C,\quad \int \sec^3(x)\ dx=\frac{1}{2}\sec(x)\tan(x)+\frac{1}{2}\ln|\sec(x)+\tan(x)|+C$$

Volume, Surface Area, and Arclength:

$$\pi \int_a^b (R^2 - r^2)\ dx, \quad 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\ dx, \quad \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$

Geometric Series: If $r \in (-1, 1)$ then, $\displaystyle\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

Taylor Series: Given f, its Taylor series centered at a is

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(n)}(a)}{n!}(x-a)^n + ...$$

Power Series: The radius of convergence R for $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ is given by

$$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \right|$$