

# 4205 Homework #2

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## 1 Part I

Here you will factor an operator and solve the equation in general by appealing to a system of first order equations.

Consider the equation

$$u_y + u_{xy} = 0 \tag{1}$$

- (a) Rewrite the equation in the form  $Xu = 0$  where  $X$  is a differential operator.
- (b) Factor your operator  $X$  into two pieces.
- (c) Write a system of 2 first order PDEs that is equivalent to equation (1)
- (d) Solve the PDE!!

## 2 Part II

Here we have fun animating waves! We will work with a model for the vertical motion of an infinitely long string:

$$\begin{aligned} u_{tt} &= 4u_{xx}; \text{ for } x, t \in \mathbb{R} \\ u(x, 0) &= \varphi(x); \text{ } x \in \mathbb{R} \\ u_t(x, 0) &= \psi(x); \text{ } x \in \mathbb{R} \end{aligned} \tag{2}$$

- (a) Suppose  $\psi = 0$  and

$$\varphi(x) = \begin{cases} 4 - x^2 & \text{for } x \in (-2, 2) \\ 0 & \text{for } x \in (-\infty, -2) \cup (2, \infty) \end{cases}$$

Using maple and our formula from class, animate the solution!

- (b) Competing waves. Repeat the animation process of the previous problem using instead:

$$\varphi(x) = \begin{cases} 4 - (x + 10)^2 & \text{for } x \in (-12, 8) \\ 4 - (x - 10)^2 & \text{for } x \in (8, 12) \\ 0 & \text{otherwise} \end{cases}$$

Include a short paragraph explaining the interaction you see.