4205 Homework #4 - Fourier Series

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1. Consider $f(x) = x^2$, $g(x) = x^4$, and $h(x) = x^3 + 1$ on $(0, \pi)$.

   (a) Draw a graph of the even, odd and periodic extensions of $f(x)$ and $h(x)$ to $(-\pi, \pi)$. Then, graph what you expect the Fourier series to converge to on the whole line $(-\infty, \infty)$.

   (b) Find the half-range even expansions for $f(x)$ and $g(x)$. Animate the convergence by animating partial sums for a bonus!

   (c) Do these series converge to $f(x)$ and $g(x)$ respectively at $x = \pi$? By which theorem?

   (d) Using your expansions at $x = \pi$, find values for the infinite sums:

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \text{ and } \sum_{j=1}^{\infty} \frac{1}{n^4}.$$

2. Here, you will demonstrate the identity

$$D_m(x) = \frac{\sin((m + 1/2)x)}{\sin(x/2)}$$

   Start with our development:

$$D_m(x) = \sum_{n=-m}^{m} e^{inx}$$

   (a) Show that

$$D_m(x) = 1 + 2 \sum_{n=1}^{m} \cos(nx)$$

   (b) Multiply both sides by $\sin(x/2)$ and use the identity

$$\cos(a)\sin(b) = \frac{\sin(a + b) - \sin(a - b)}{2}$$

   to rewrite your series.

   (c) Your result in the previous step produces a telescoping sum. Show that it indeed adds up to $\frac{\sin((m + 1/2)x)}{\sin(x/2)}$.

   (d) Using this last formula, plot $D_m(x)$ and animate the picture as m increases from 1 to 50.