1206 HW#2 Solutions.

#1. (i) $A = \int_4^9 \left(x^{1/2} + 2x\right) dx$

$$= \left(\frac{2}{3}x^{3/2} + x^2\right)\Big|_4^9$$

$$= \frac{2}{3}\left(9^{3/2} - 4^{3/2}\right) + \left(9^2 - 4^2\right)$$

$$= \frac{2}{3}(27 - 8) + (81 - 16)$$

$$= \frac{38}{3} + 65$$

$$= \frac{233}{3}.$$

(ii) $\int_0^1 \frac{1}{1+t^2}\, dt$

$$= \tan^{-1}(t)\Big|_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}.$$

(iii) $\int_0^{\pi} \sin\left(\frac{z}{2}\right) dz$ Let $u = \frac{z}{2}$.

Then, $2du = dz$

$$= 2\int_0^{\pi/2} \sin(u)\,du$$

$$= -2\cos(u)\Big|_0^{\pi/2}$$

$$= -2(0) + 2(1)$$

$$= 2.$$

#2. $G(s) = \int_a^s x^4\,dx$

(i) $G(s) = \frac{x^5}{5}\Big|_a^s = \frac{s^5}{5} - \frac{a^5}{5}$.

(ii) Since $G'(s) = \frac{d}{ds}\left(\frac{s^5}{5} - \frac{a^5}{5}\right) = \frac{5s^4}{5} = s^4$

we see $G'(s) = f(s)$ and so G is an A.D. of f.

(iii) $A = \int_a^b x^4 dx = G(b)$ and so,

$$A = \frac{b^5}{5} - \frac{a^5}{5}.$$

#3. a) $\int \sin(x)\cos(x)dx$

Set $u = \sin(x)$

$du = \cos(x)dx \longrightarrow \int \sin(x)\cos(x)dx$

$$= \int u\,du = \frac{u^2}{2} + C$$

$$= \frac{\sin^2(x)}{2} + C.$$

b) $\int t^2 (t^3+2)^{32} dt$

$u = t^3 + 2 \longrightarrow \int t^2 (t^3+2)^{32} dt$

$\frac{1}{3} du = t^2 dt$

$$= \frac{1}{3}\int u^{32} du = \frac{1}{3(33)} u^{33} + C$$

$$= \frac{1}{99}(t^3+2)^{33} + C.$$

c) $\int \frac{\ln(t)}{t}\,dt$ $\quad u = \ln(t)$

$du = \frac{1}{t}\,dt$

$= \int u\,du$

$= \frac{u^2}{2} + C = \frac{\ln^2(t)}{2} + C.$

d) $\int w e^{-w^2}\,dw$ $\quad u = e^{-w^2}$

$du = -2we^{-w^2}\,dw$

$-\frac{1}{2}du = we^{-w^2}\,dw$

note:
$u = -w^2$
also works!

So, $\int we^{-w^2}\,dw = -\frac{1}{2}\int du$

$= -\frac{1}{2}u + C = -\frac{1}{2}e^{-w^2} + C.$

e) $\int \frac{\sqrt{x}}{x^{3/2}+1}\,dx = \int \frac{x^{1/2}}{x^{3/2}+1}\,dx$

Set $u = x^{3/2} + 1$.

Then $du = \frac{3}{2} x^{1/2} dx$

giving $\frac{2}{3} du = x^{1/2} dx$.

We set:

$$\int \frac{x^{1/2}}{x^{3/2}+1} dx = \frac{2}{3} \int \frac{1}{u} du$$

$$= \frac{2}{3} \ln |u| + C$$

$$= \frac{2}{3} \ln |x^{3/2}+1| + C$$

(*) On the domain of $h(x) = x^{3/2} + 1$, $h(x) \geq 0$ always. So, we can remove the abs. value and write

$$\int \frac{\sqrt{x}}{x^{3/2}+1} dx = \frac{2}{3} \ln\left(x^{3/2}+1\right) + C.$$