Math 1206 Class Test #1 - Practice Test

This is meant for practice and so there are more problems here than on your actual test. Your test will consist of 7 problems only and, each problem will have fewer parts.

(1) Evaluate each of the following integrals and indicate the technique you use if applicable.

(a) $\displaystyle\int \left(x^{-3/2} + 3x^{\pi} - 6\right)\, dx$

(b) $\displaystyle\int t^3 \ln(t^4 + 1)\, dt$

(c) $\displaystyle\int (s + 1)e^s\, ds$

(d) $\displaystyle\int \sin^2(s)\cos^3(s)\, ds$

(e) $\displaystyle\int \frac{1}{x\,(\ln(x))^4}\, dx$

(f) $\displaystyle\int \frac{y}{4 + 3y}\, dy$

(g) $\displaystyle\int \left(1 - \frac{u^2}{1 + u^2}\right)\, du$

(2) Draw a picture of the region R below the function $f(x)$ and above the indicated interval on the x-axis. Following this, calculate the area of R.

(a) $f(x) = x\ln(x)$ over $[1, e]$.

(b) $f(x) = \begin{cases} e^x & \text{if} \quad 0 \leq x \leq 1 \\ e^x + 1 & \text{if} \quad 1 < x \leq 2 \end{cases}$ over $[1, 2]$.

(3) Consider $f(x) = \tan(x)\sec^2(x)$ on the interval $[0, \pi/4]$ and define $g(s) = \int_0^s f(x)\,dx$.

(a) Using the substitution $u = \tan(x)$, evaluate $g(s)$.
(b) Using the substitution $u = \sec(x)$, evaluate $g(s)$.
(c) Are your results the same? If not, explain why this is this case.

(4) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_0^{e^2} x(\ln(x))^2 \, dx$

(b) $\displaystyle\int_0^{\infty} \frac{e^x}{1+e^{2x}} \, dx$

(c) $\displaystyle\int_0^{1} \frac{1}{\sqrt{x}} \, dx$

(5) Consider the sequence $\left\{ \frac{n^2}{\frac{1}{2}+n^4} \right\}_{n=1}^{\infty}$.

(a) Write the first 4 terms of the sequence.
(b) Is the sequence increasing or decreasing? Justify.
(c) Does the sequence converge? If so, to what limit?
(d) Decide if $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\frac{1}{2}+n^4}$ converges or diverges. Justify.

(6) Consider the sequence $\left\{\frac{e^n-1}{e^n}\right\}_{n=1}^{\infty}$.

(a) Decide if the sequence is increasing or decreasing.

(b) Is the sequence bounded above or below?

(c) Using a theorem from class, decide if the sequence converges or diverges without evaluating a limit.

(7) (a) Find the sum of the series $\sum_{n=1}^{\infty}\left[\left(\frac{2}{7}\right)^n - \frac{2}{7}\left(\frac{2}{7}\right)^n\right]$.

(b) Find a formula for the partial sum

$$S_k = \sum_{n=1}^{k}\left[\frac{1}{n} - \frac{1}{n+1}\right].$$

Using the formula you found, find $\sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{1}{n+1}\right]$.

(8) Using convergence tests, decide which of the following series converge or diverge. **Indicate the test you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{4+n^6}$.

(b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$.