1206 HW #4 Solutions

#1.

(a) $\{j\}_{j=1}^{\infty} = \{1, 2, 3, 4, 5, 6 \ldots\}$

The sequence diverges as $\lim_{j\to\infty} (j) = \infty$

(b) $\left\{\frac{1}{2n}\right\}_{n=1}^{\infty} = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots\right\}$

Since $\lim_{n\to\infty} \frac{1}{2n} = 0$, the sequences converges to 0.

(c) $\left\{\frac{12n^3-1}{1-7n^2+8n^3}\right\}_{n=1}^{\infty}$.

Since $\lim_{n\to\infty} \frac{12n^3-1}{1-7n^2+8n^3} = \frac{12}{8} = \frac{3}{2}$,

the sequence converges to $3/2$.

#2. a) $\{-2, -1, 4, 7, 10, \dots\}$

Notice that the elements increase by 3 for each increment in n. More, we get every element by adding enough 3's!

$$a_1 = -2 = -2 + 0(3)$$

$$a_2 = -1 = -2 + 3$$

$$a_3 = 4 = -2 + 2(3)$$

$$\vdots$$

$$a_n = -2 + (n-1)\cdot 3 = 3n - 5.$$

Our sequence is $\{3n-5\}_{n=1}^{\infty}$ and it diverges to ∞.

b) $\left\{ \frac{3}{2}, \frac{5}{4}, \frac{9}{8}, \frac{17}{16}, \dots \right\}$

Here $a_1 = \frac{3}{2} = \frac{2^1+1}{2^1}$

$$a_2 = \frac{5}{4} = \frac{2^2+1}{2^2}$$

$$a_3 = \frac{9}{8} = \frac{2^3+1}{2^3}$$

and so on. So, $a_n = \dfrac{2^n+1}{2^n}$

and our sequence is:

$$\left\{ \frac{2^n+1}{2^n} \right\}_{n=1}^{\infty}$$

It converges to 1.

#3. a) $\displaystyle\sum_{n=1}^{\infty} \tan^n(\pi/3)$.

$\tan\frac{\pi}{3} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$. So, our series is

$$\sum_{n=1}^{\infty} (\sqrt{3})^n$$ which is a divergent geometric series as

$r = \sqrt{3} > 1$.

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

Notice that $\frac{1}{n^2+n} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

So, our series is the telescoping infinite series $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$. The sequence of partial sums $\{b_k\}_{k=1}^{\infty} = \left\{\sum_{n=1}^{k}\left(\frac{1}{n} - \frac{1}{n+1}\right)\right\}_{k=1}^{\infty}$ can be written as

$$b_k = \sum_{n=1}^{k}\left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{k+1}$$

and so $\lim_{k\to\infty} b_k = 1$ giving

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n} = 1.$$